If Louis Pasteur was correct that chance favors the prepared mind, then it found the perfect candidate in Edward Norton Lorenz, MIT mathematician and meteorologist and father of chaos theory, a science many now believe rivals even relativity and the quantum in importance.

Photo: Massachusetts Institute of Technology

The moment came one winter day 1961 at MIT. Lorenz was running a climate model consisting of twelve differential equations representing climate parameters when he decided to reexamine one the run's sequences. From printout he took conditions from a mid-point in the model run and reinitiated the calculations, making only one slight change: The original inputs had six decimal-digits, and Lorenz, to save time and space, rounded them to three for the second run. He quite reasonably expected that his second run would precisely match hiss first, but it didn't. It was, in fact, almost precisely the same at the beginning, but then the second run diverged radically, bearing no resemblance to its mathematical parent. Clear that something was wrong, Lorenz first suspected a hardware problem, but there nothing amiss with his Royal McBee computer's vacuum tubes. Then Lorenz realized the truth. The rounding of the initial inputs—a tiny change in initial values—had produced wildly divergent results.

Long-term weather forecasting was doomed, Lorenz realized, because of the climate's "sensitive dependence on initial conditions." He described it as "The Butterfly Effect"—a perfect choice of terms given the graphic the Lorenz strange attractor, with its fractal dimension, generates. The implications of Lorenz's discovery—the chaotic nature of climate—are staggering. Human tampering with with climate's atmospheric gases, the melting its glaciers and ice caps and the resultant loss of albedo, the temperature of the oceans, and changes to innumerable other factors can wreak havoc on our climate, engendering changes in weather as yet unimagined. Lorenz armed us with the predictive capability to understand the potential impact of global climate futures that we may inadvertently create—or consciously decide to prevent.

And what of the Lorenz strange attractor itself? Both Dr. Marcelo Viana—winner of the Ramanujan Prize and Grand Croix of the Order of Scientific Merit awarded by the President of Brazil—and Fields Medal winner Dr. Jean-Christophe Yoccoz have studied the Lorenz Strange Attractor in multiple dimensions. Research on the Lorenz strange attractor continues on the most advanced edges of mathematical inquiry.

In awarding Edward Norton Lorenz the 1991 Kyoto Prize—one of a plethora of honors and awards bestowed upon him—the Inamori Foundation wrote: "He made his boldest scientific achievement in discovering 'deterministic chaos,' a principle which has profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind's view of nature since Sir Isaac Newton."

On the following pages Dr. Timothy Palmer, Head of the Probability Forecast Division at the European Centre for Medium-Range Weather Forecasts and Dr. Clint Sprott, an award-winning lecturer and author of *Chaos and Time-Series Analysis,* pay tribute to Dr. Lorenz work—and explain its components, meaning, and implications.

Dr. Lorenz is a currently a professor emeritus at the Massachusetts Institute of Technology.

$$\frac{dy}{dt} = \sigma(y - x) \qquad \frac{dx}{dt} = \rho x - y - xz \qquad \frac{dz}{dt} = xy - \beta z$$

In these equations, σ is the Prandtl number (which represets the ratio of fluid viscosity to its thermal conductivity; ρ represents the temperature difference between the top and the bottom of the system; and β is the ratio of width to height of the box used to enclose the system. [See "Strange Attractors" at http://www.pha.jhu.edu/~ldb/seminar/attractors.html]

Strange Attractors

Photo © Jeff Miller

"Lorenz saw the possibility of more complicated attractors that were neither stable points nor periodic cycles, and his great achievement was to show not only that such attractors exist, but that they can arise from very simple mathematical models. Hence the irregularity and unpredictability of the weather is not necessarily a consequence of the complexity of the governing equations but is an inherent property of the system."

In the early 1960's Edward Lorenz, a young meteorology professor at MIT, had what was surely one of the first personal computers, although you would hardly recognize it as such. He was using it to understand why the weather has such erratic fluctuations, despite the regular diurnal and seasonal variations in sunshine. In particular, he was using his computer to solve a simple set of equations that model atmospheric convection, hoping to find solutions that were not periodic. His success was accompanied by an unexpected discovery—sensitive dependence on initial conditions—which he dubbed the 'butterfly effect,' since such behavior in the atmosphere would make long-range weather prediction impossible. His toy equations produced the Lorenz attractor, a geometrical object that serendipitously resembles the wings of a butterfly, and thus became an emblem of the modern chaos era.

I could have been at the forefront of that movement since I was a physics student at MIT taking classes and doing research just a few hundred feet from where Lorenz was then working, but it was another twenty-five years before I became aware of his work, and forty years later that I

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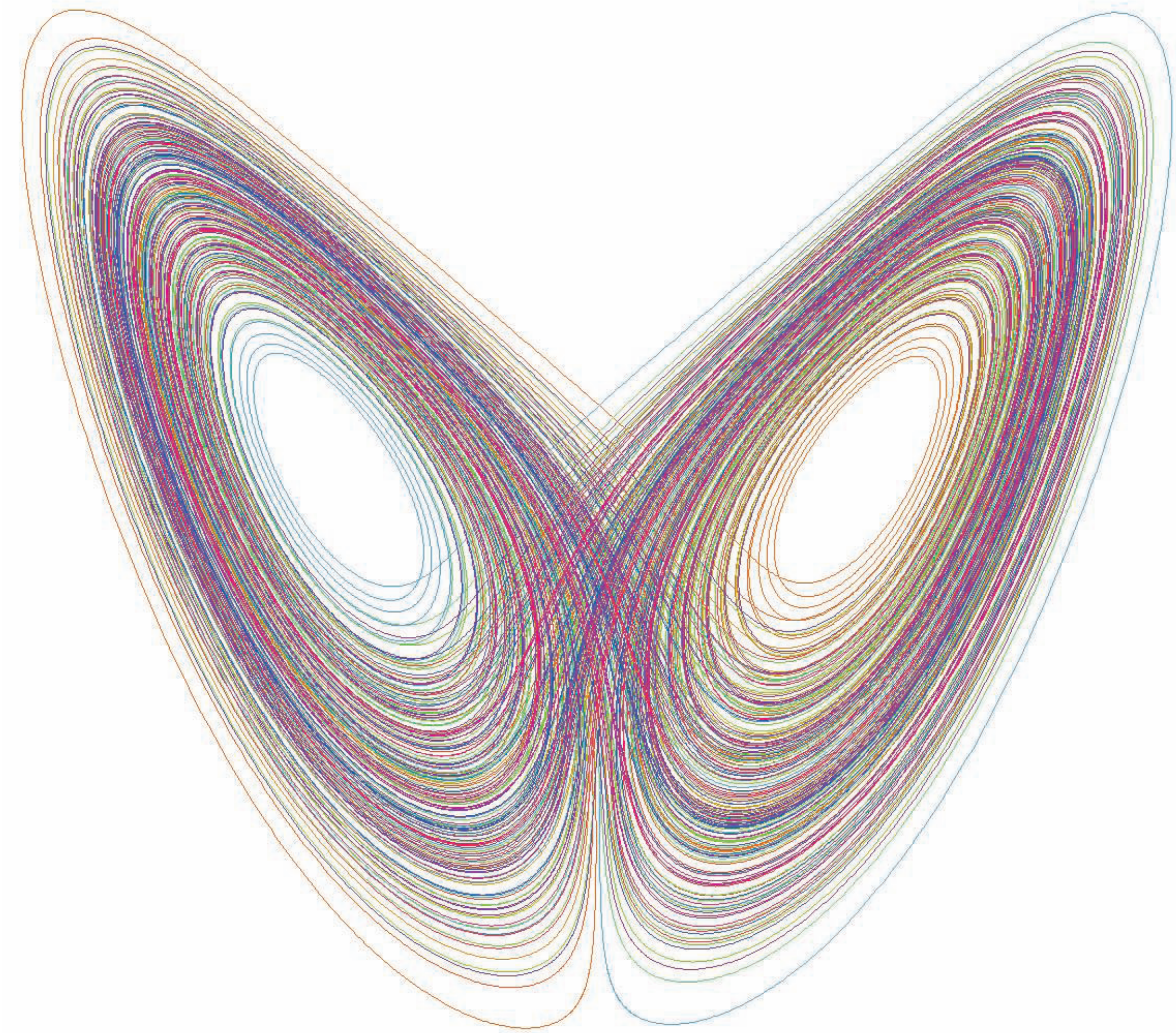
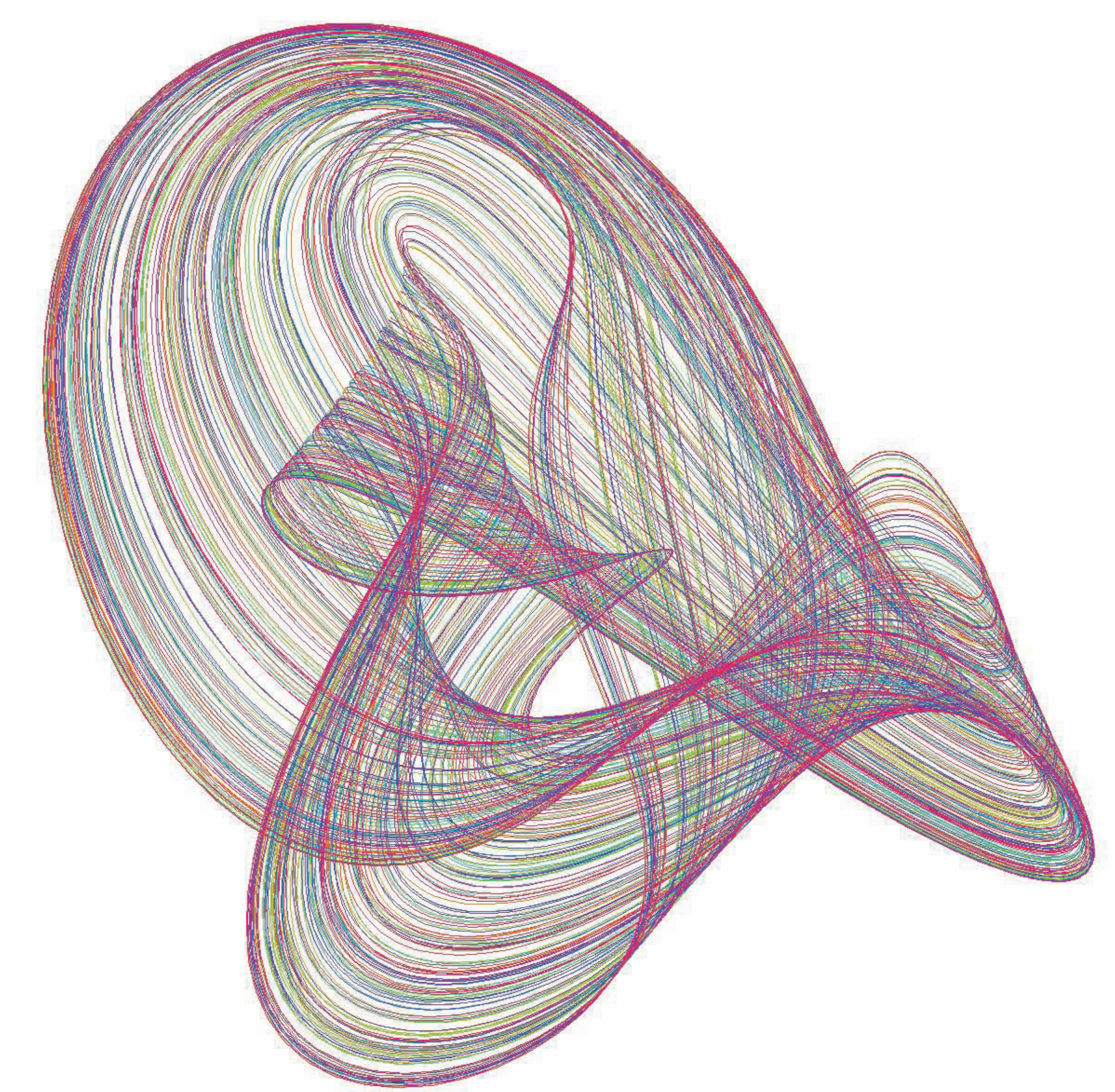
had a chance to meet him. How could it have escaped my attention that such simple equations can have such complicated solutions? Perhaps chaos is very rare, as suggested by the fact that everyone was studying a handful of examples that were known in the 1980's.



The Lorenz Fractal. Model: Clint Sprott

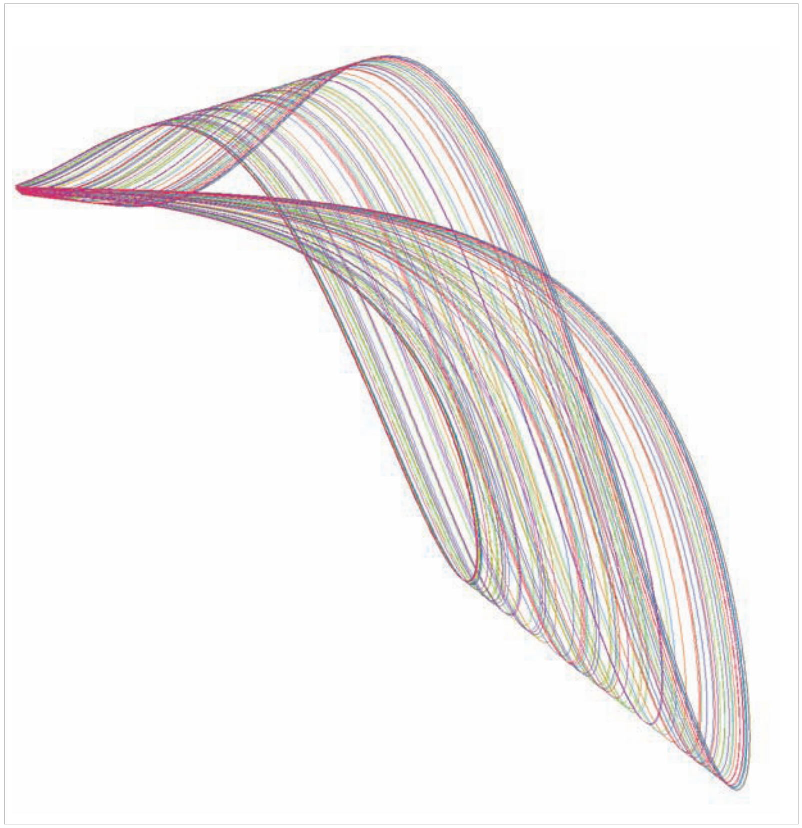
I decided to automate the search for chaos in systems of equations and began finding thousands of new examples, each producing an object that David Ruelle and Floris Takens called a 'strange attractor,' some from equations even simpler than those used by Lorenz, and many with great aesthetic appeal. Shown on the next page is a sample of the 62 such objects in the Appendix of my chaos textbook. Hundreds more, along with a simple explanation of the math and science behind them are contained in the coffee-table art and poetry book that I wrote with Robin Chapman.

So what are these gracefully swirling strange attractors? To understand them, it is useful to consider the kinds of attractors that were known prior to Lorenz's discovery. The simplest attractor is a point (a single dot), and it represents a stable equilibrium. In a model of the weather, it would mean that the temperature and other conditions in a particular location is the same day after day forever, unless there was some external disturbance such as a volcano erupting, in which



Halvorsen's Cyclically Symmetric Attractor. Model: Clint Sprott

case the conditions would depart from the equilibrium but would eventually return to it. A two-dimensional plot showing the temperature at two different locations would wiggle around but would seem to be attracted to the equilibrium point in the aftermath of any such disturbance.



The Sprott Simplest Chaotic Flow, Discovered by Clint Sprott

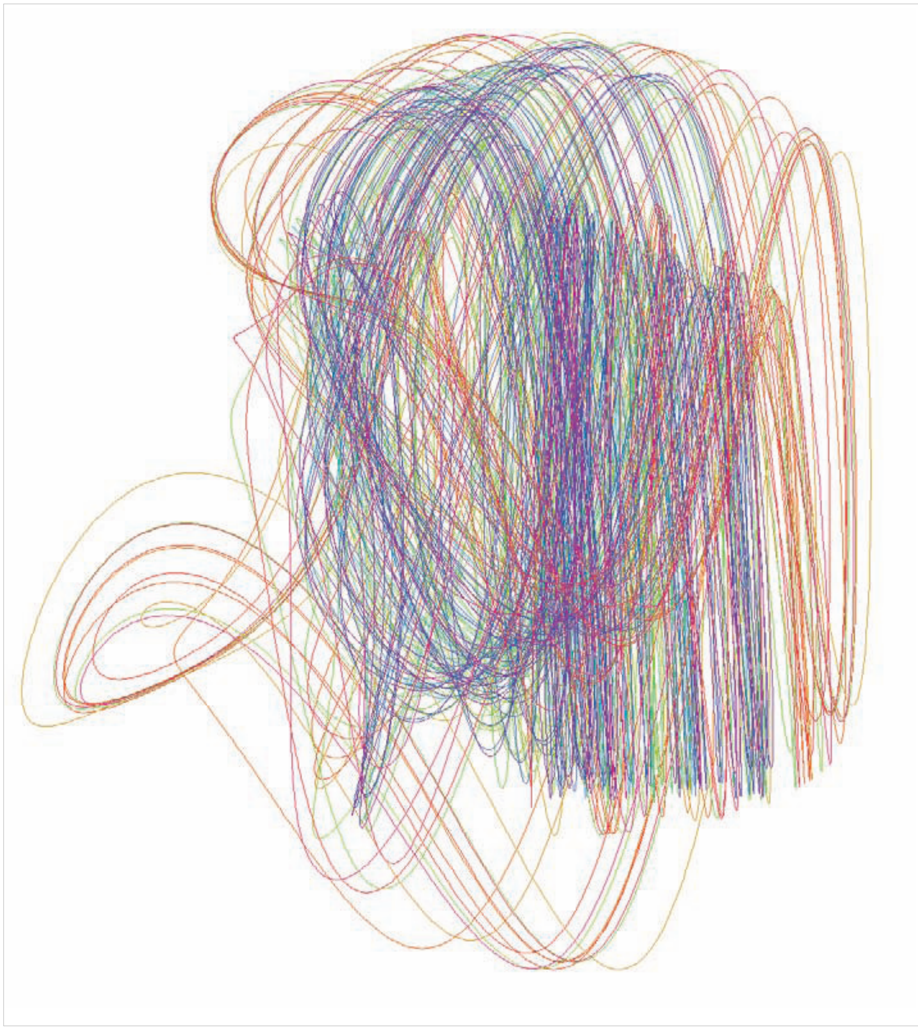
Clearly the weather is not and cannot be a point attractor. Instead, the temperature should rise during the day and fall at night as solar heating comes and goes. Thus one might expect a plot of the temperature at two locations a continent apart to cycle daily around a closed loop, and if this cycle is stable, it is called a 'limit cycle,' which is a one-dimensional attractor. Considering also the seasonal cycle and locations at different latitudes, one might expect a more complicated, perhaps two-dimensional attractor but still consisting of some number of regular periods.

Lorenz saw the possibility of more complicated attractors that were neither stable points nor periodic cycles, and his great achievement was to show not only that such attractors exist, but that they can arise from very simple mathematical models. Hence the irregularity and unpredictability of the weather is not necessarily a consequence of the complexity of the governing equations but is an inherent property of the system, and many dynamical systems in diverse fields such as ecology and economics may share this property.

In a chaotic system, the trajectory moves around on the attractor as time goes on, but two nearby points separate exponentially so that eventually they are very far apart. Although their future is determined uniquely and precisely by the governing equations, very small differences in the starting point can make large differences in the future conditions. Although tomorrow's weather depends on the conditions today, and the weather the day after tomorrow depends on the conditions tomorrow, small errors in measuring the current weather eventually grow until all hope of predictability is lost — the 'butterfly effect.'

Strange Attractors

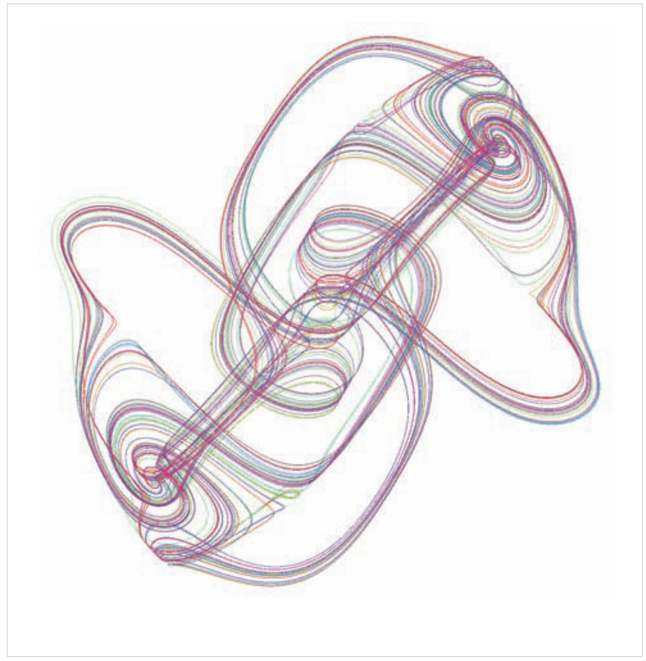
If the Lorenz attractor is neither a point, nor a line, nor a surface, what is it? It is a geometrical object called a 'fractal' that has structure on all scales and a dimension that is not an integer. For example, the Lorenz attractor has a dimension (by one method of calculation) of 2.06215. Such an object is almost a surface (dimension 2.0), but it cannot be flattened out, and it must live in a space of at least three dimensions. In fact, systems like the weather in which the variables change continuously (as opposed to abruptly), can only be chaotic if they have three or more variables. The atmosphere is actually an infinite-dimensional system, consisting of several variables such as the temperature at infinitely many spatial locations, and so it is hardly surprising that it would behave chaotically, and its attractor certainly has a much higher dimension than does the Lorenz attractor.



The Hadley Circulation Fractal. Model: Clint Sprott

In a chaotic system, the trajectory moves around on the attractor as time goes on, but two nearby points separate exponentially so that eventually they are very far apart. Although their future is determined uniquely and precisely by the governing equations, very small differences in the starting point can make large differences in the future conditions. Although tomorrow's weather depends on the conditions today, and the weather the day after tomorrow depends on the conditions tomorrow, small errors in measuring the current weather eventually grow until all hope of predictability is lost — the 'butterfly effect.'

Thus Lorenz showed that long-range weather prediction will always be impossible, although we might be able to better characterize the attractor and understand how it might change as parameters in the equations change. In particular, there can be abrupt changes in the behavior at 'bifurcation points,' more popularly called 'tipping points.' Furthermore, the very behavior that makes prediction difficult makes control possible. If a butterfly flapping its wings can cause a tornado, we can hope that small modifications to the environment might prevent catastrophic events like tornadoes, hurricanes, and perhaps even global warming. Who would have thought that a single individual working with a primitive computer could have precipitated such a radical paradigm shift? That will be the legacy of Edward Lorenz.



Thomas' Cyclically Symmetric Attractor.
Model: Clint Sprott

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